Motion Detection Using Dynamic Mode Decomposition

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Introduction

- **Dynamic Mode Decomposition (DMD)** is a simple, interpretable data-driven technique to disentangle space-time dynamics
- Provides a linear approximation to a dynamical system
- Input: snapshots arranged in two **data matrices**

$$X = [X_1 \quad X_2 \quad \cdots \quad X_n],\ Y = [Y_1 \quad Y_2 \quad \cdots \quad Y_n],\ Y_i = X_{i+1}$$

- **DMD matrix** $A = YX^{\dagger}$ solves:

$$\underset{A \in \mathbb{C}^{M \times M}}{\operatorname{argmin}} \|Y - AX\|_F^2$$

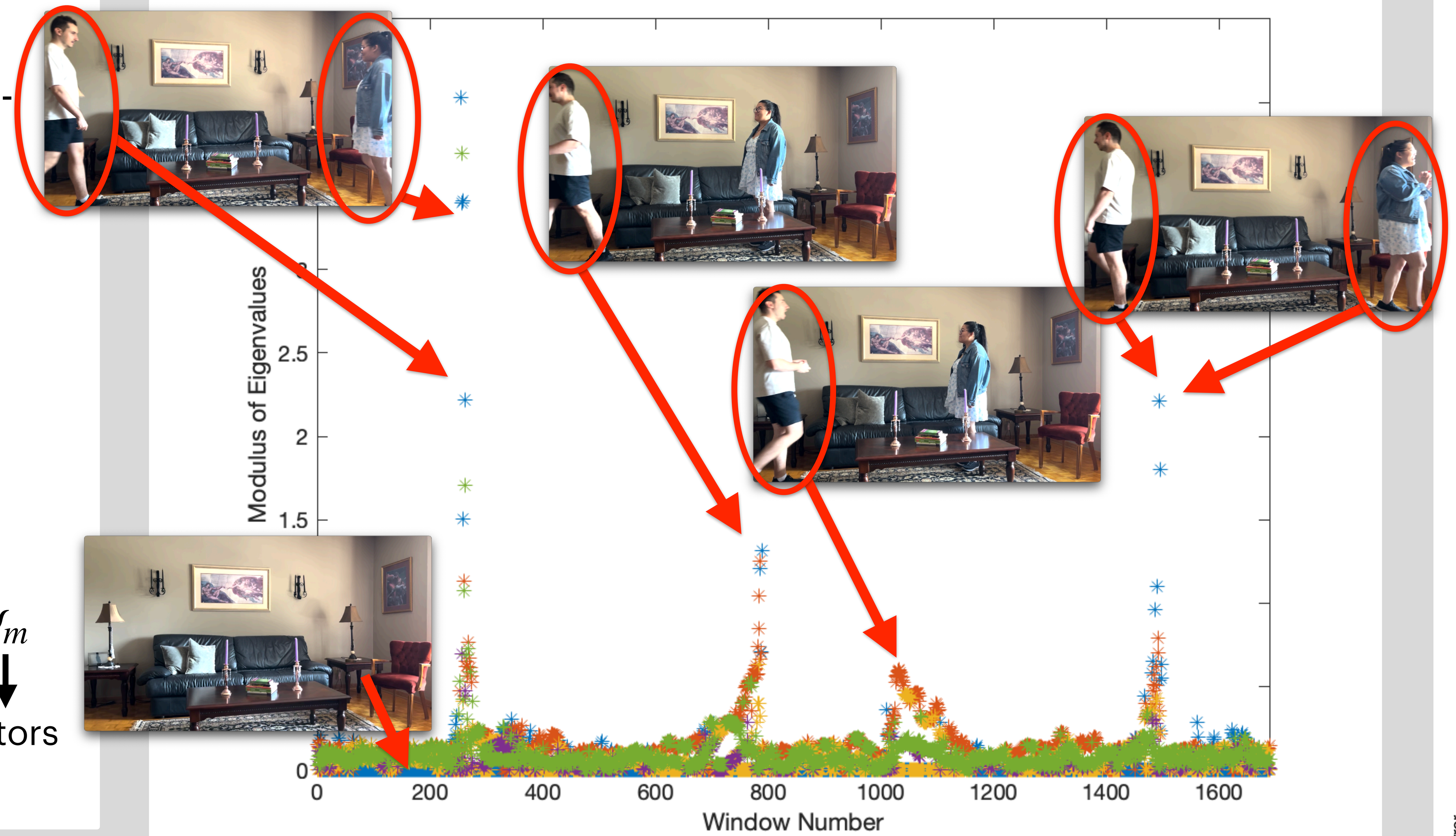
If A is diagonalizable, we can decompose: $X_k = \sum_{m=1}^{M} c_m \lambda_m^k \psi_m$

Eigenvalues

Eigenvectors

Objective: leverage this to implement an automatic motion detection system for video

Motion Detection Scheme



Eigenvalue average in each window

$$\mu^{(k)} = \frac{1}{r}\sum_{i=1}^{r} |\omega_i^{(k)}|$$

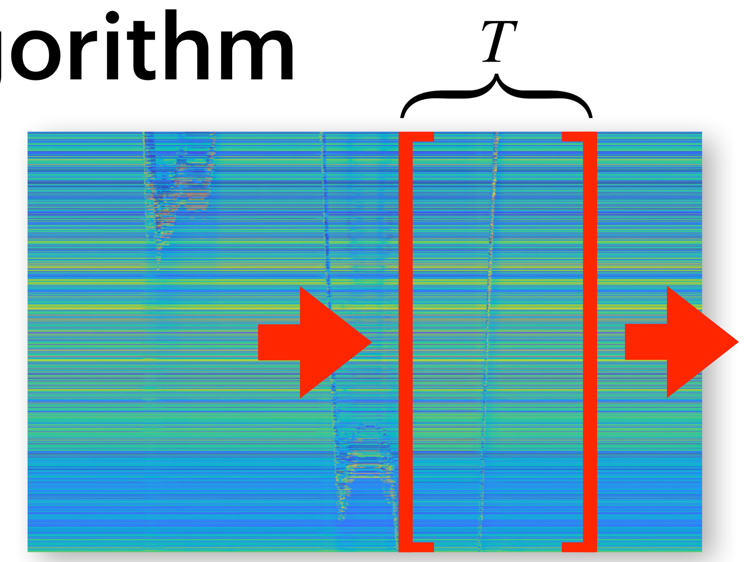
Compute relative change

$$\Delta = \frac{|\mu^{(k+1)} - \mu^{(k)}|}{|\mu^{(k)}|}$$

$\Delta > \Delta^*$ → Motion Detected!

Optimizing Δ^* requires an error measure: $E = FP + c \cdot FN$

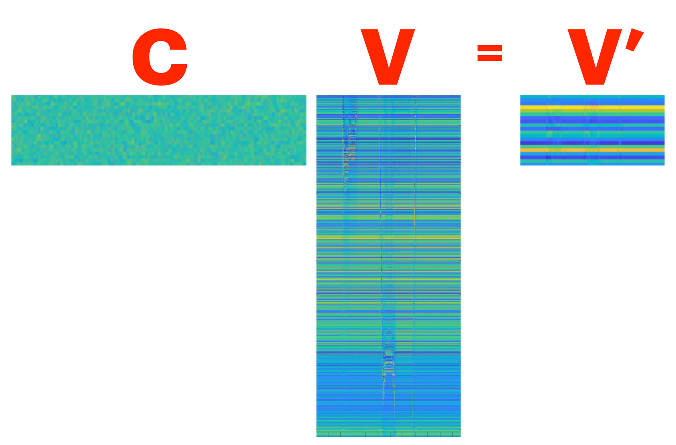
DMD Algorithm



Input: video matrix $V \in \mathbb{R}^{M \times N}$

1. Compressed DMD

High dimensional video data V is compressed using a random matrix $C : V' = CV$



2. Sliding Window DMD

Streaming data is separated into windows of size T to apply DMD

3. Rank 'r' Approximation

Take the SVD of compressed data matrix X' to find best rank r approximation: $X' \approx U_r \Sigma_r V_r^*$

4. Rank Reduced DMD Matrix

Calculate the projection of the DMD matrix A onto U_r:

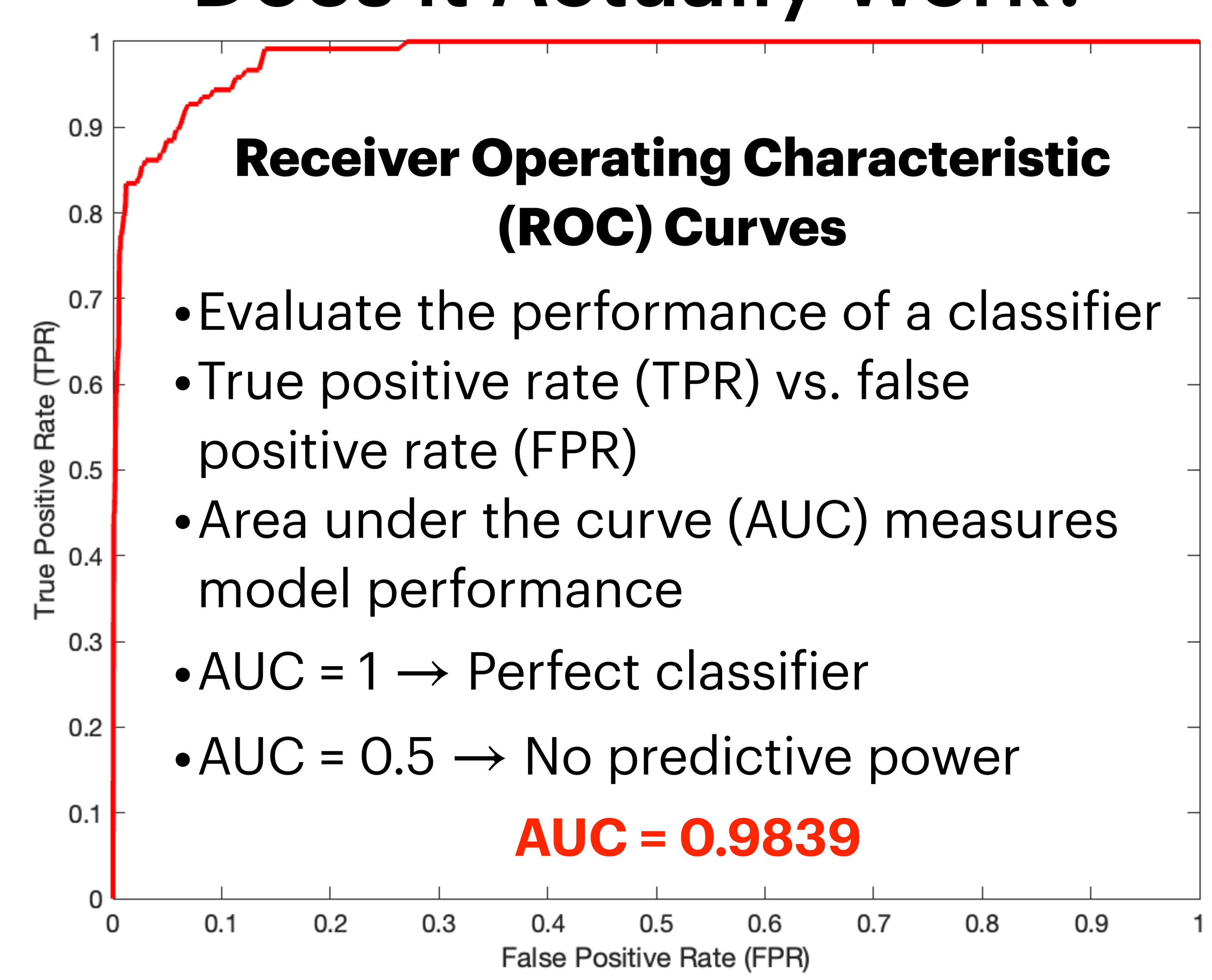
$$\tilde{A} = U_r^* A U_r = U_r^T Y_2 V_r \Sigma_r^{-1}$$

5. Extract Eigenvalues

$$\tilde{A}v = \lambda v \implies \omega = \log \lambda / \Delta t$$

Output: matrix of eigenvalues

Does it Actually Work?



Receiver Operating Characteristic (ROC) Curves

- Evaluate the performance of a classifier
- True positive rate (TPR) vs. false positive rate (FPR)
- Area under the curve (AUC) measures model performance
- AUC = 1 → Perfect classifier
- AUC = 0.5 → No predictive power

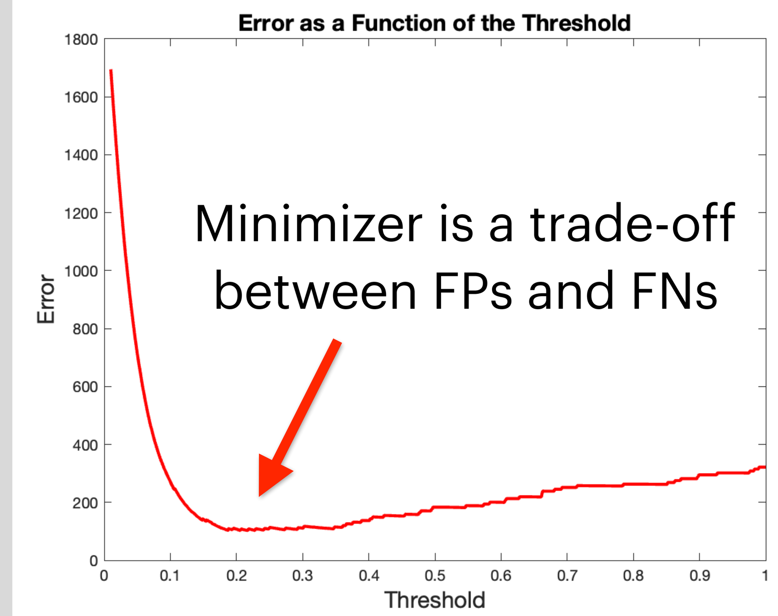
AUC = 0.9839

Event Detection Training

Pseudo k-fold Cross Validation

1	V	T	T	T
2	T	V	T	T
3	T	T	V	T
4	T	T	T	V

1. Shuffle the given dataset randomly
2. Split into k groups (**folds**) of equal size
3. For k iterations
   1. Separate into **training set (T)** and **validation set (V)**
   2. Train the model on T, tracking error for each Δ^*
   3. Threshold with smallest average error is optimal
   4. Test the threshold value on V and compute error
5. Threshold with lowest overall error is optimal



Results

		Iter 1	Iter 2	Iter 3	Iter 4
Easy	Threshold	0.450	0.348	0.410	0.348
	Avg. err.	17.00	31.33	33.67	68.33
Hard	Threshold	0.348	0.348	0.360	0.450
	Avg. err.	91.00	29.00	28.80	127.00

Conclusion

We have proposed a simple, effective, and interpretable method to detect movement in video

- Method works best when subjects are clear and isolated
- Future work: comparison with standard motion detection methods and training with a larger video database

References

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